



Connectivity bottlenecks in ocean flows characterized by the Lagrangian betweenness

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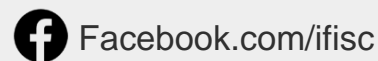
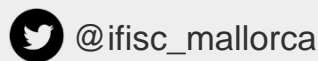
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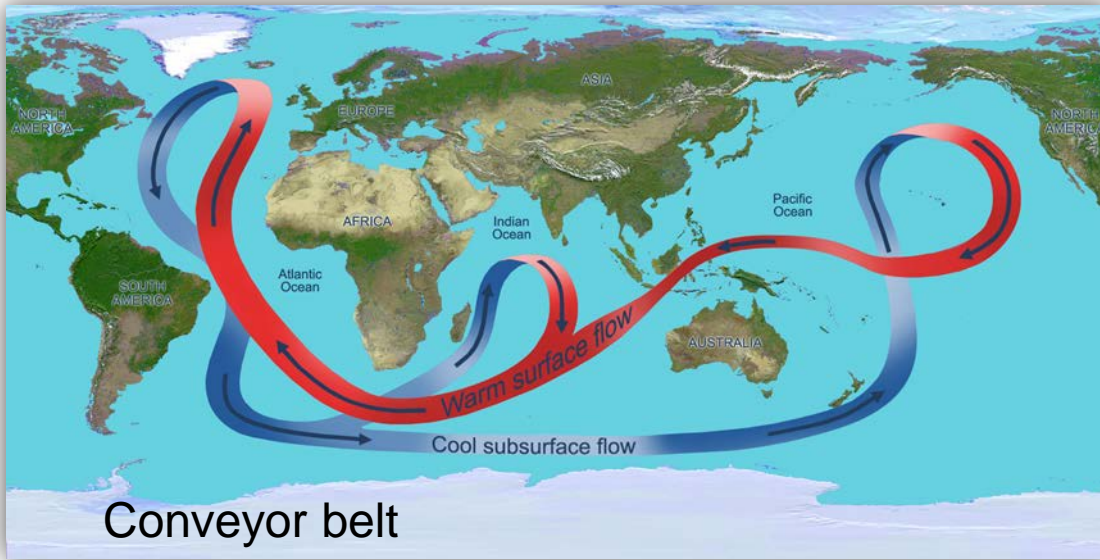
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OUTLINE

- Lagrangian Flow networks: network representation of fluid transport.
- Betweenness: a quantification of bottlenecks in networks and flows.
- Lagrangian betweenness: betweenness from Lyapunov exponents
- Bottlenecks in the ocean
- Conclusions



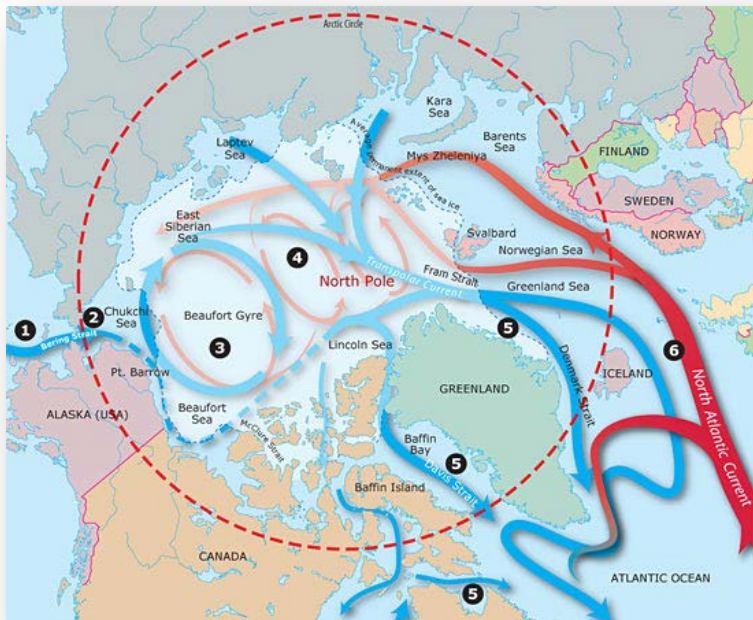
FLUID FLOWS: TRANSPORT OF WATER, MOMENTUM, HEAT, SUBSTANCES, BIOTA...

TRANSPORT: BARRIERS AND CONNECTIVITY

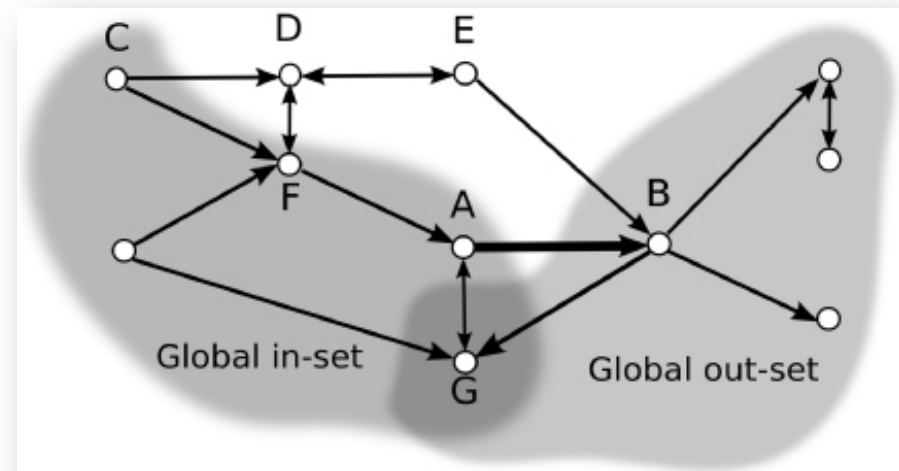
FLUID FLOW AS TRANSPORT NETWORKS

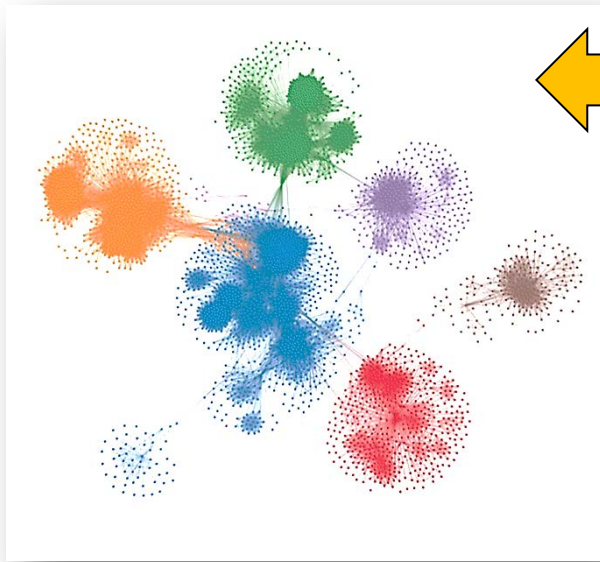
Lagrangian Flow Networks

Ser-Giacomi et al. (2015) Chaos 25, 036404

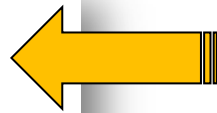


Powerful techniques of network theory become available



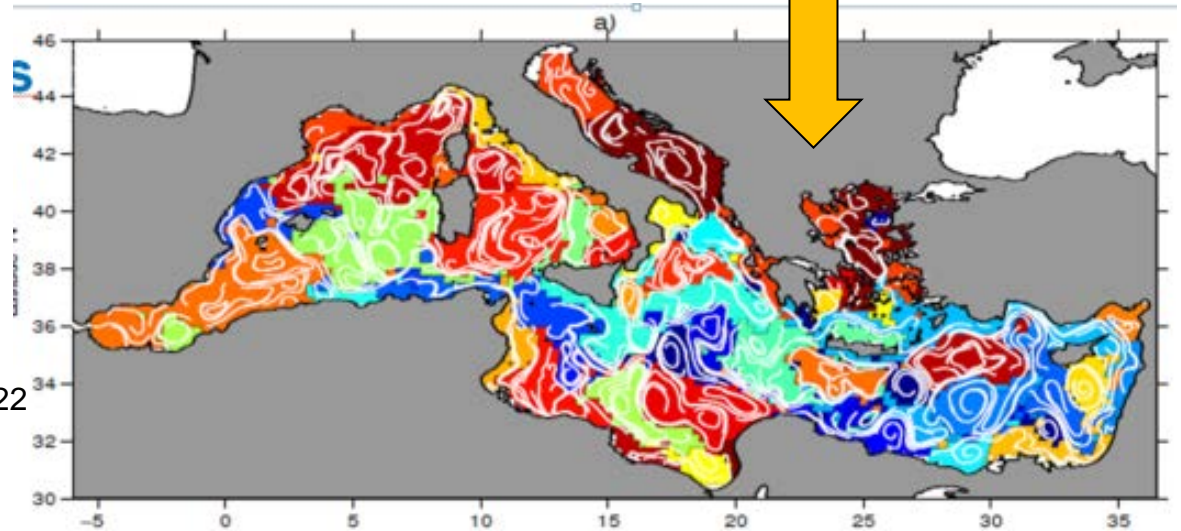


Facebook network, Fortunato & Newman, Nature 2022

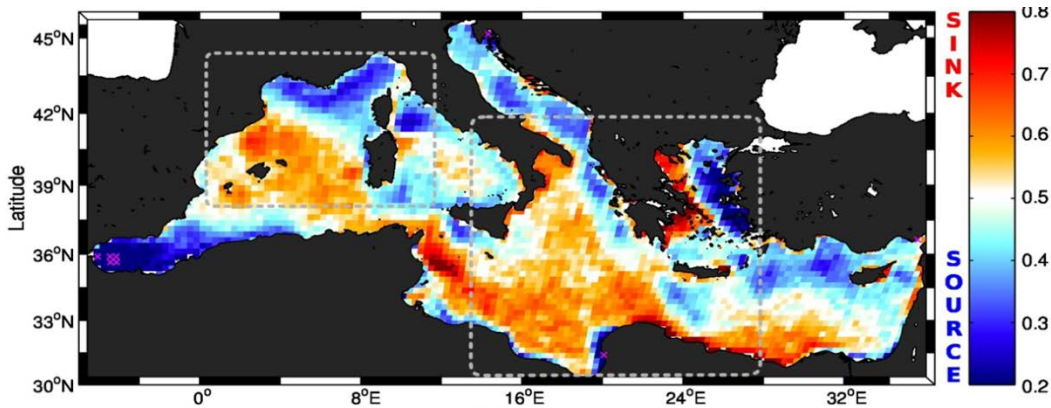


PARTITION IN COMMUNITIES

MARINE PROVINCES

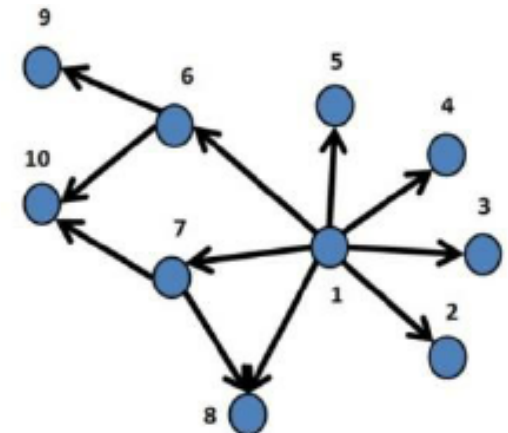


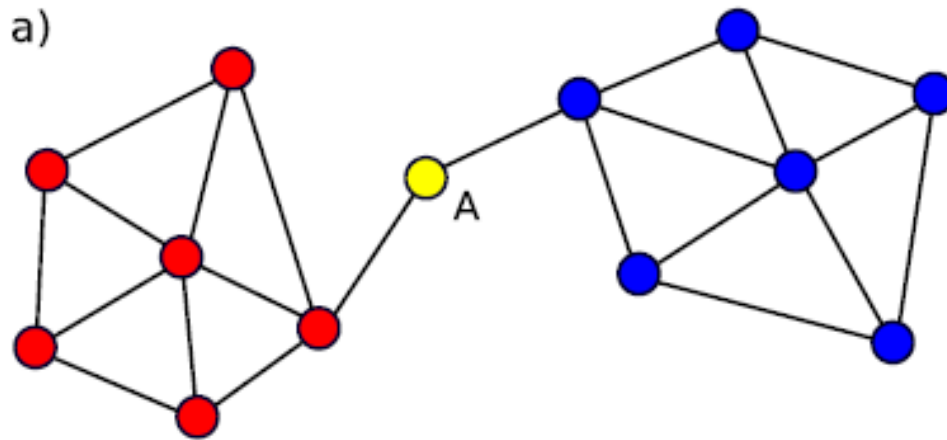
Ser-Giacomi et al. (2015), Chaos 25, 087413, Rossi et al. (2014), Geophys Res Lett 41, 2883



Dubois, Rossi, Ser-Giacomi, Arnaud-Haond, Lopez, Hernandez-Garcia, Global Ecology and Biogeography 25, 503 (2016)

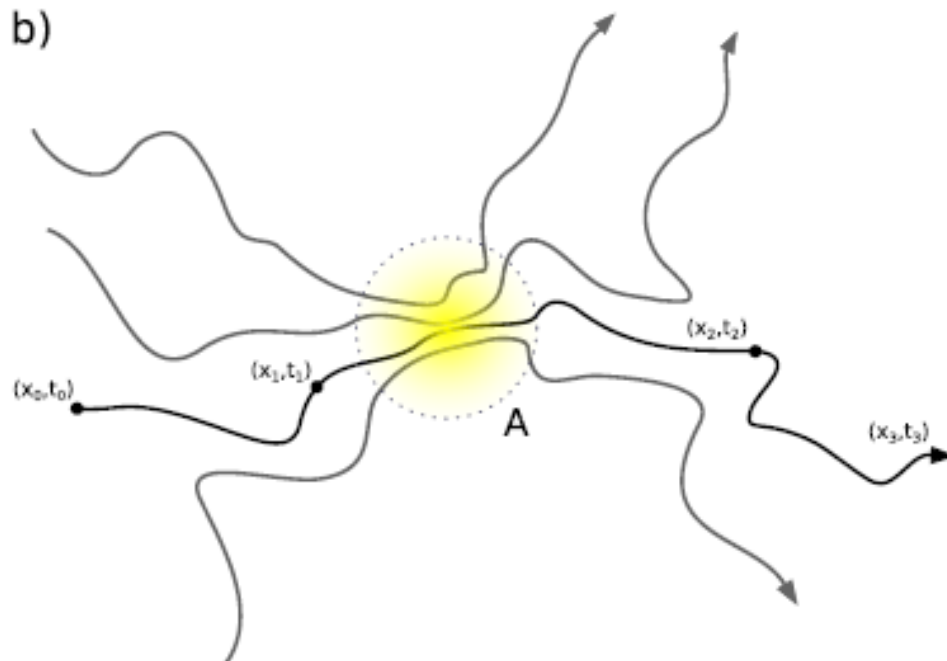
**SOURCES
AND SINKS**





Betweenness in networks:

B_i = Proportion of optimal paths (shorter, faster, most probable, maximum flow, ...) crossing node i among all optimal paths between all pairs of nodes



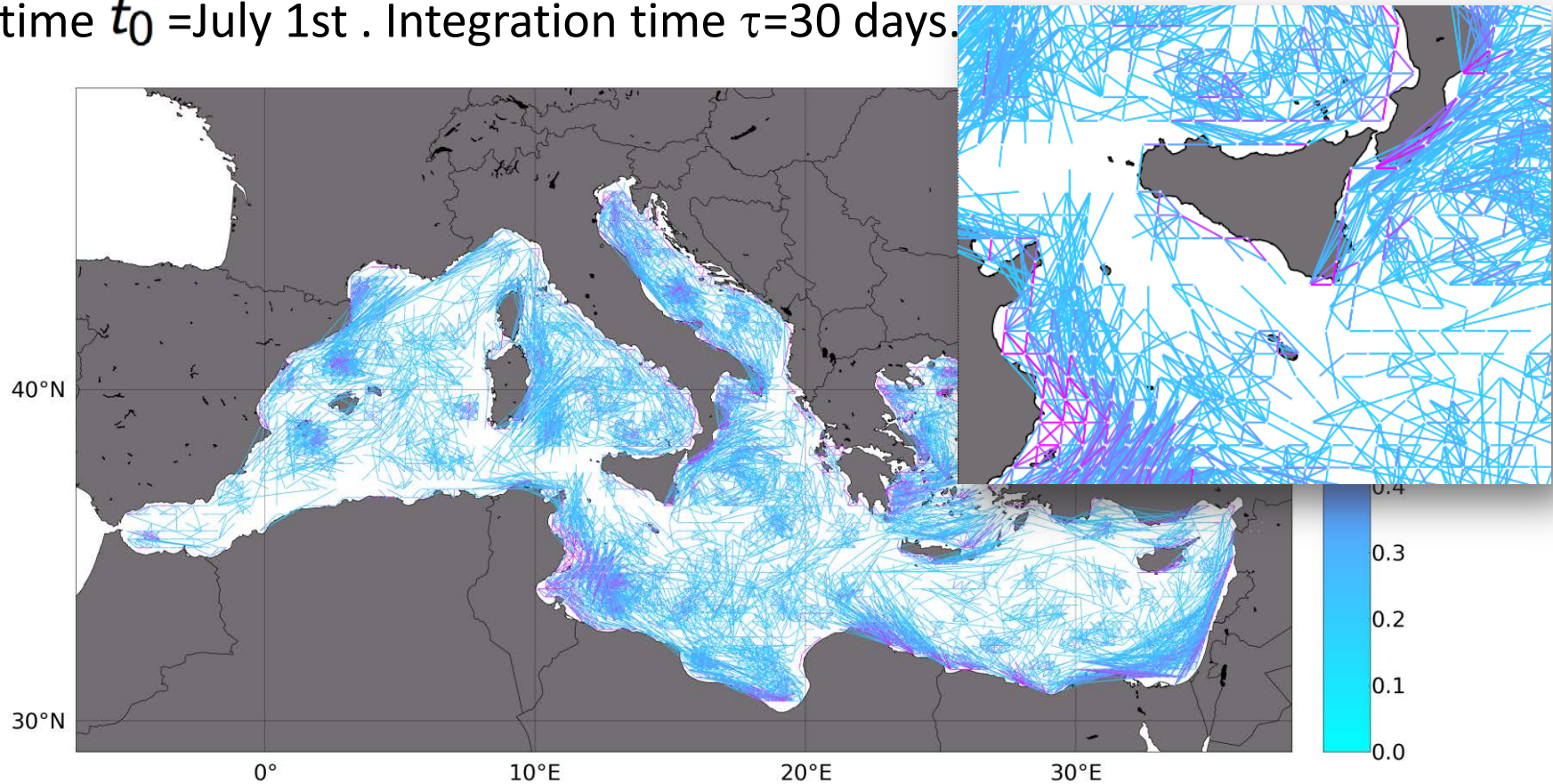
Betweenness in flow networks or dynamical systems:

Flow bottlenecks

Importance: mixing, dispersion, ecological hot spots, ...

Example of LAGRANGIAN FLOW NETWORK construction

start at time t_0 = July 1st . Integration time τ = 30 days.



Lagrangian Flow network

$$P_{ij}(t_0, \tau)$$

Probability for a Lagrangian tracer to be in location j after a time τ if starting in location i at time t_0 .



STANDARD CONSTRUCTION OF BETWEENNESS IN FLOW NETWORKS

Example: Surface circulation in the Mediterranean
(2002-2011)

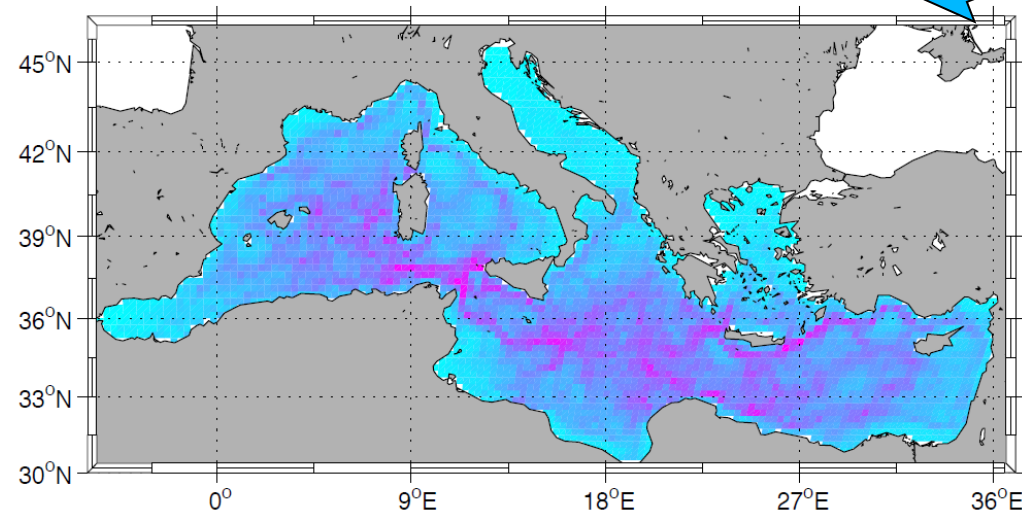
Ser-Giacomi et al, Physical Review E **92**, 012818 (2015)

Lagrangian Flow network

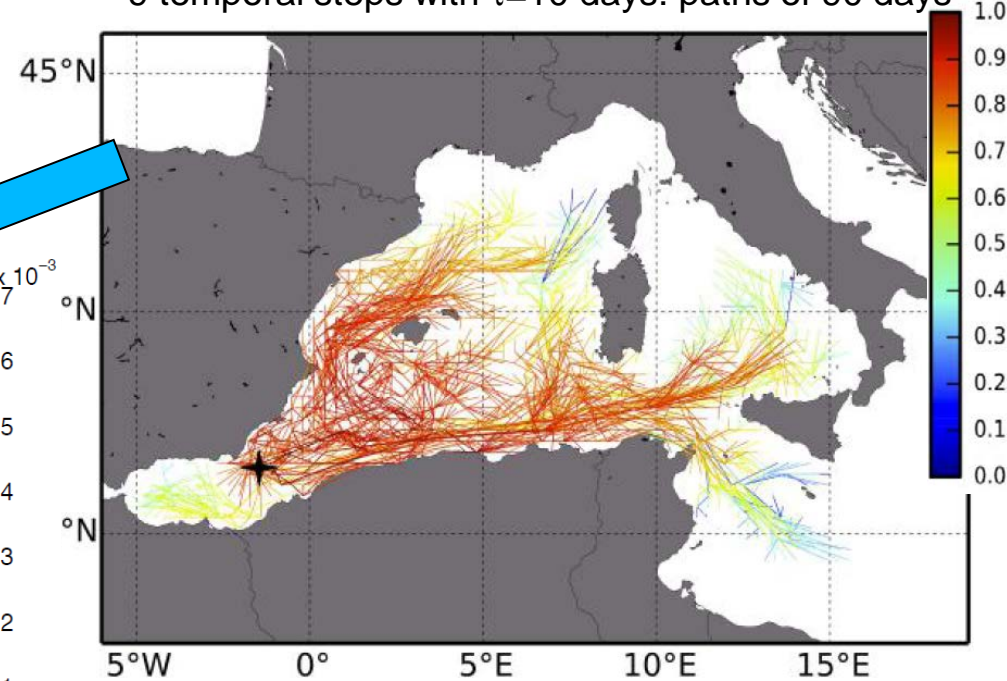
$$P_{ij}(t_0, \tau)$$

$t_0 = \text{January 1st 2011}$

9 temporal steps with $\tau=10$ days: paths of 90 days



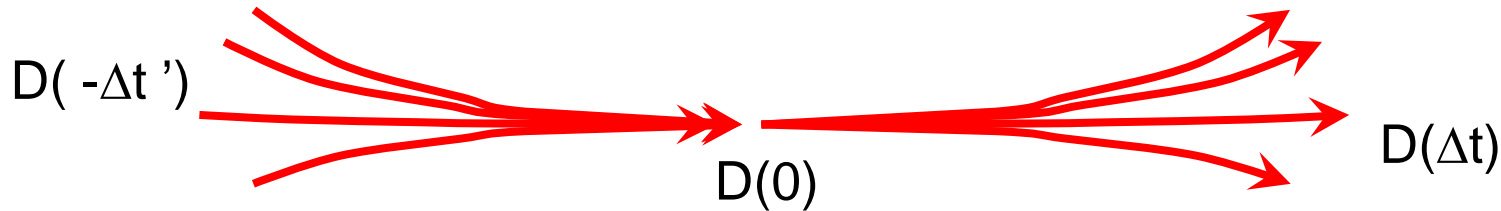
MPP-Betweenness: bottlenecks of the flow
(2002-2011)



Most-probable paths between locations.
Dijkstra-type algorithms in
Ser-Giacomi et al, Chaos **25**, 087413 (2015),
Physical Review E **92**, 012818 (2015)

In fluids or dynamical systems –

(Finite Time)Lyapunov exponent: rate of divergence of close trajectories



$$D(-\Delta t') = D(0) \exp[\lambda(\mathbf{x}_0, \mathbf{t}_0, -\Delta t') \Delta t'] \quad D(\Delta t) = D(0) \exp[\lambda(\mathbf{x}_0, \mathbf{t}_0, \Delta t) \Delta t]$$

This motivates: Definition of **LAGRANGIAN BETWEENNESS**
AT POINT \mathbf{x}_i AND TIME $t_0=0$, for trajectories of duration τ :

$$B_i^L(0, \tau) = \frac{1}{\tau} \int_0^\tau e^{t\lambda(\mathbf{x}_i, t, -t)} e^{(\tau-t)\lambda(\mathbf{x}_i, t, \tau-t)} dt.$$

Ser-Giacomi, et al, Nature Comm. 12:4935 (2021)

Computationally easier, numerically similar to MPP-betweenness

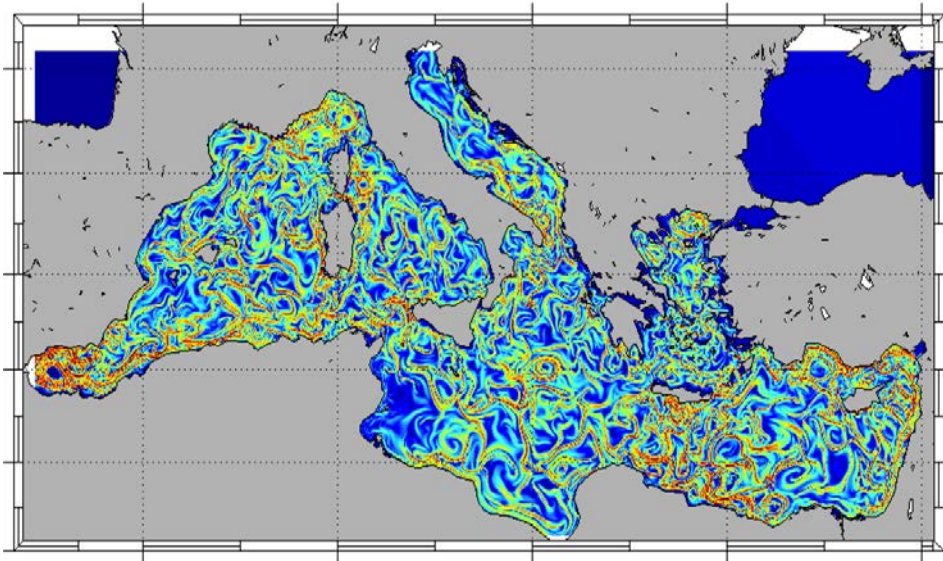
In Lagrangian Flow networks, quantitative relationship between

Finite-time (local) Lyapunov exponent $\lambda(\mathbf{x}_0, \mathbf{t}_0, \tau)$
and **out-degree** $K_{out}(i)$

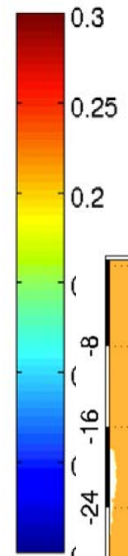
(Ser-Giacomi et al. Chaos 25, 036404 (2015))

$$\left\langle e^{\tau\lambda(\mathbf{x}_0, \mathbf{t}_0, \tau)} \right\rangle_{B_i} \approx K_{out}(i)$$

LYAPUNOV EXPONENT FIELDS IN SURFACE CIRCULATION

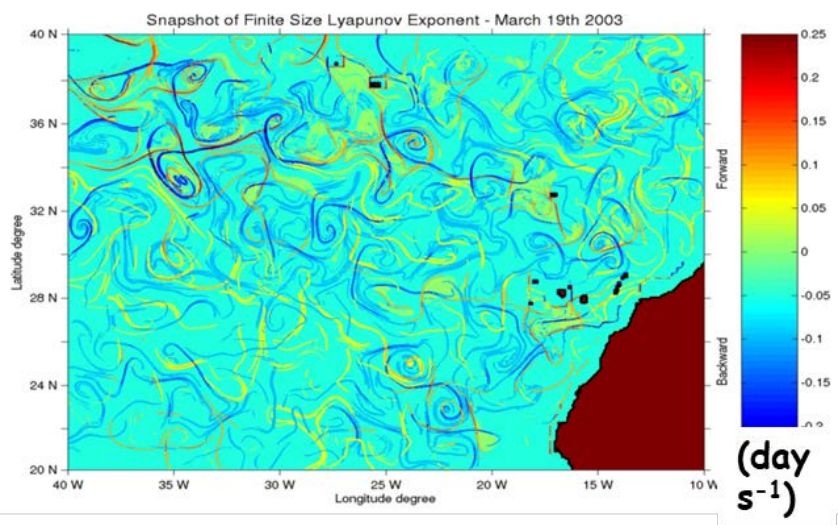
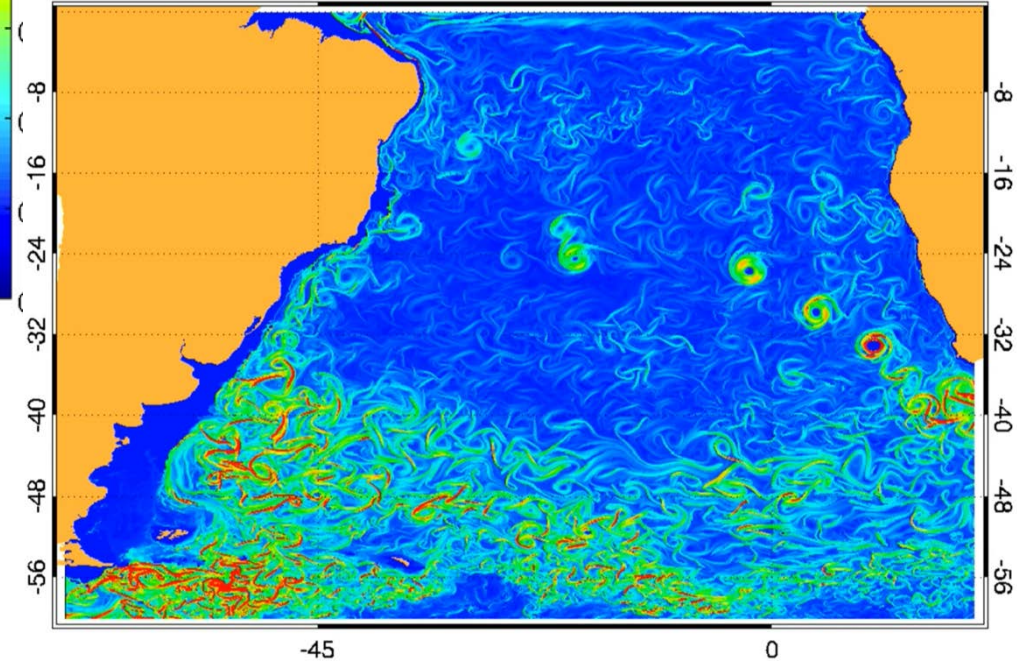


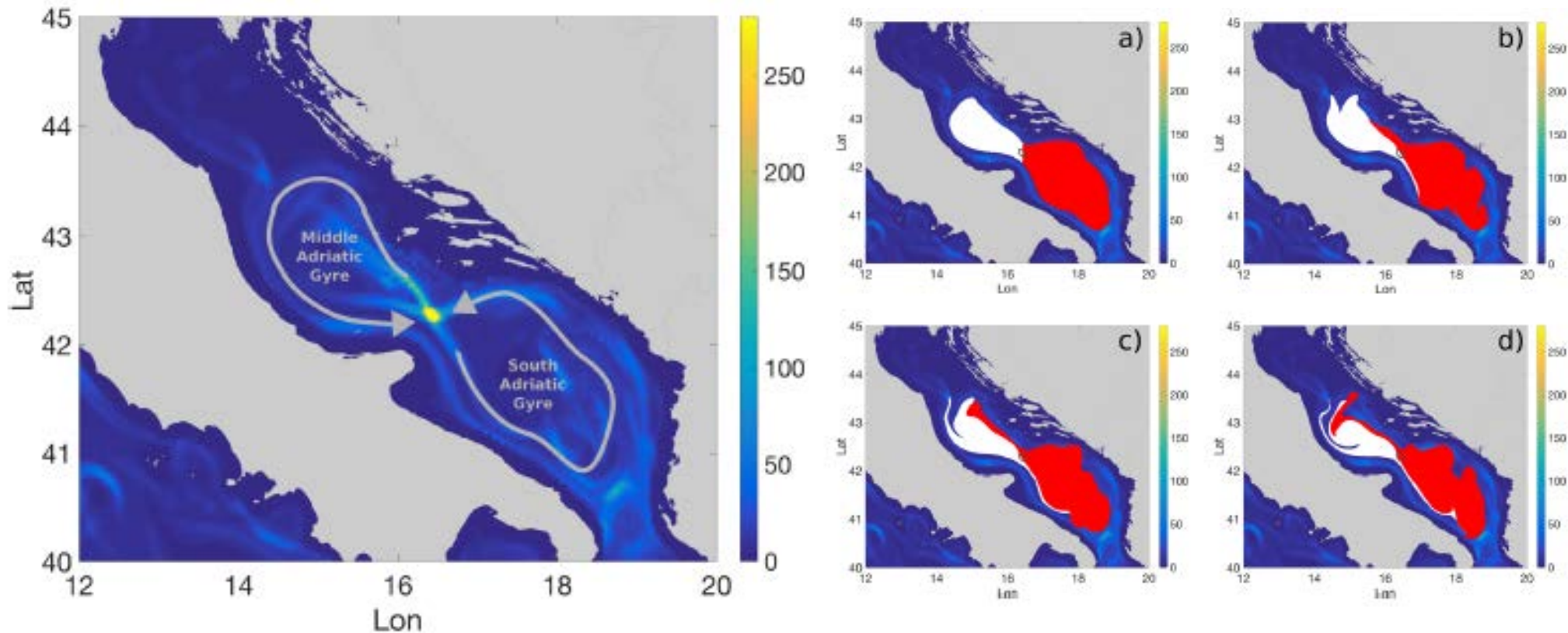
Ser-Giacomi et al. (2015), *Chaos* 25, 087413,



Input: velocity fields from simulations, reanalysis or altimetry

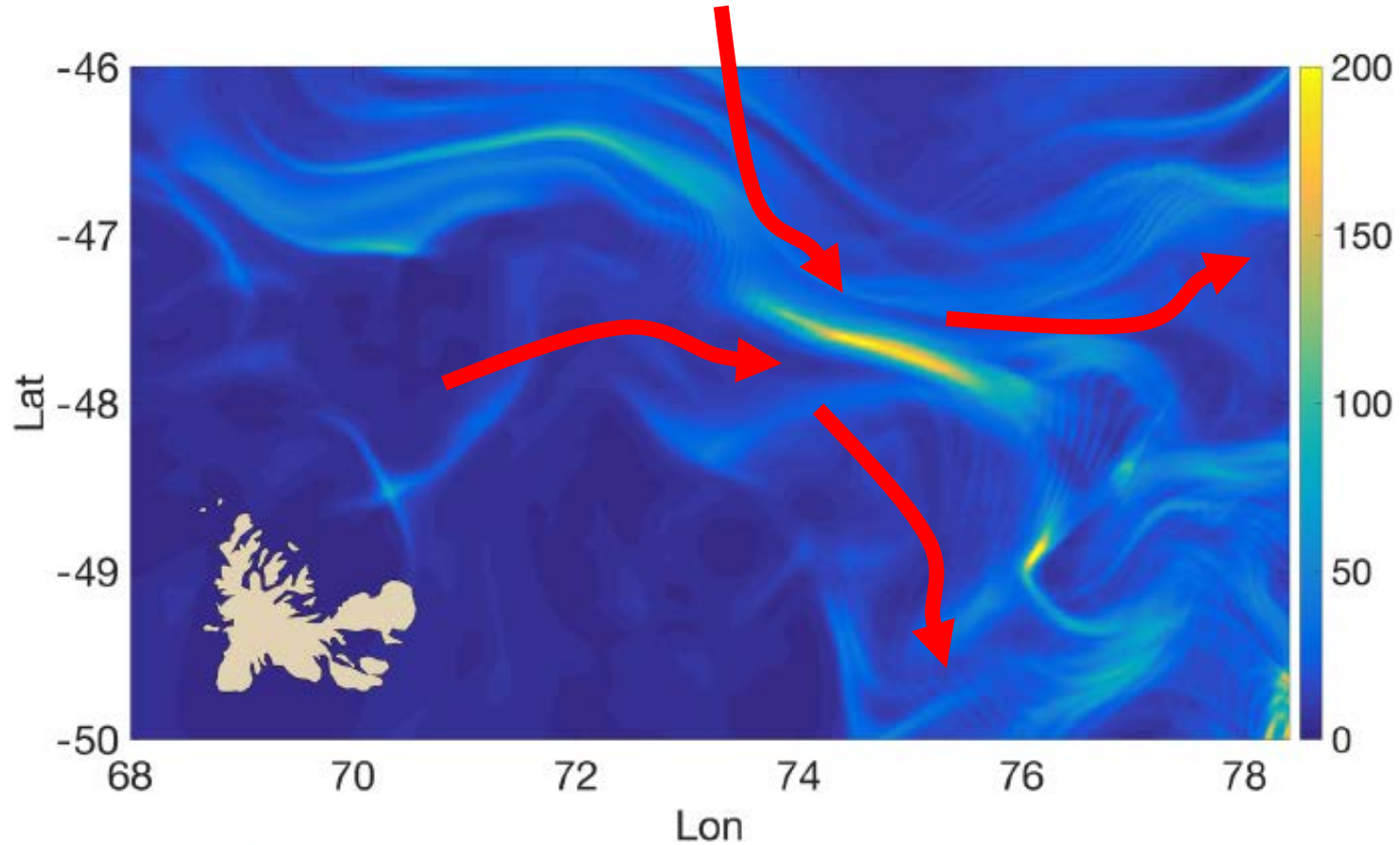
Hernández-Carrasco, López, Hernández-García, Turiel, , *J. Geophys. Res.* **117**, C10007 (2012)





Lagrangian betweenness in the Adriatic sea (13 Dec. 2013. $\tau = 15$ days). Currents's data from Med Copernicus reanalysis at $(1/16)^\circ$.

Time evolution of (red and white) water masses, to show the bottleneck character of the high-betweenness zones.



Kerguelen region (Southern ocean), $\tau=20$ days, December 2007

Velocities from SSALTO/DUACS Altimetry

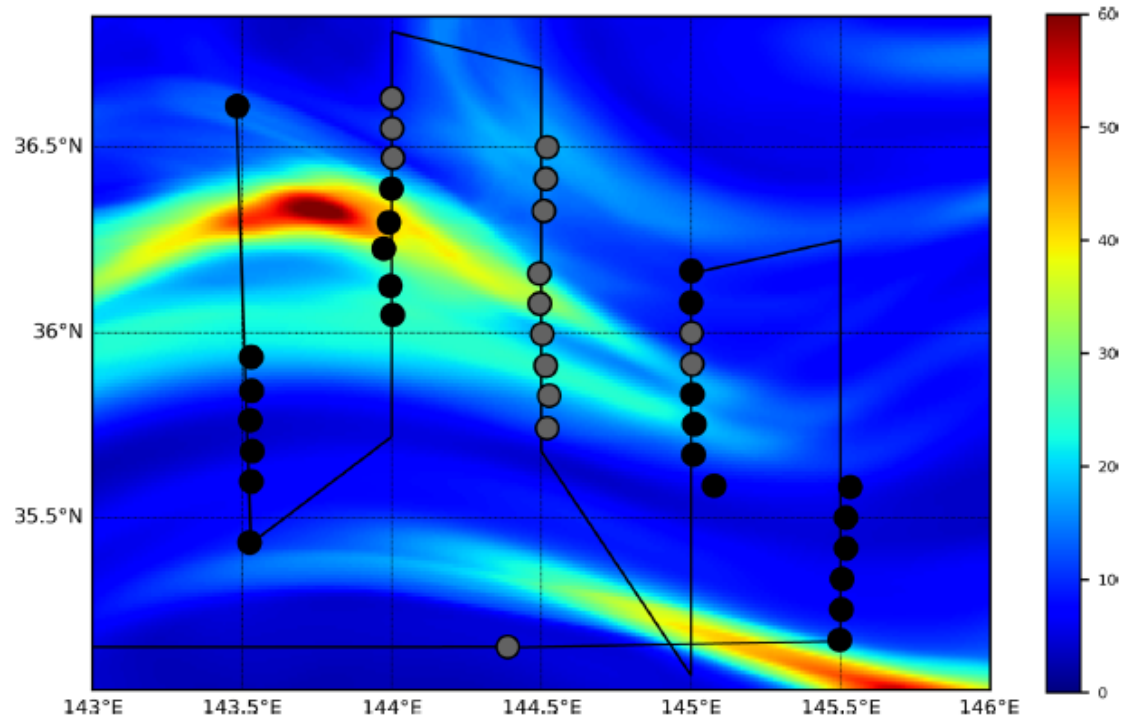


Betweenness and biodiversity

Betweenness in the Kuroshio extension front (North Pacific). Velocities from GLORYS12V1 reanalysis. 20 October 2009, $\tau=7$ days

Cruise measurements of planktonic diversity

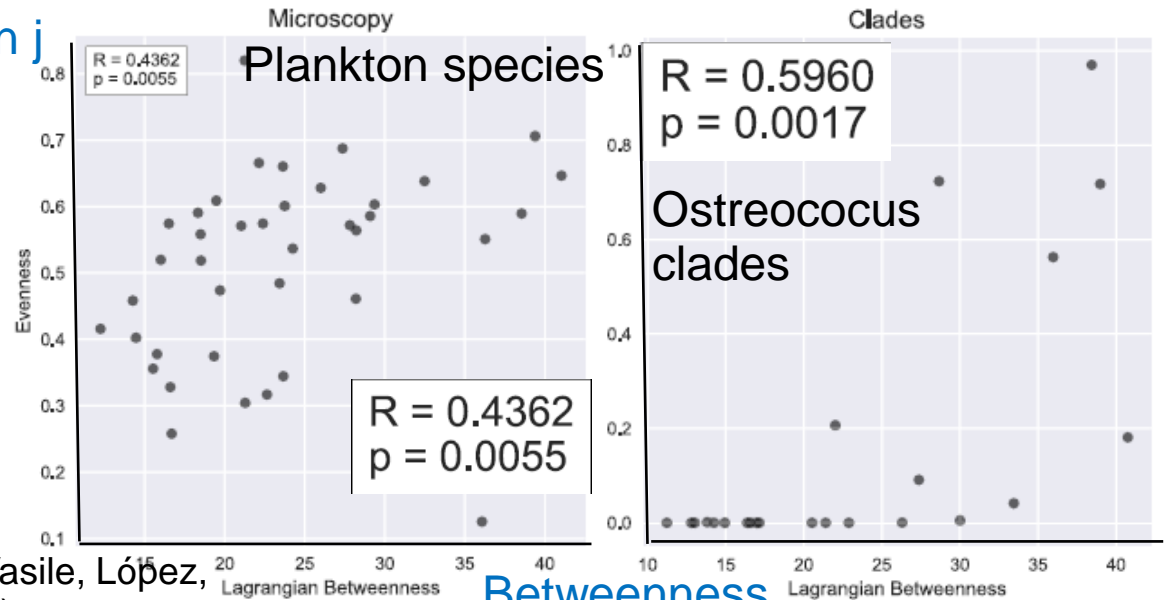
- Microscopy-based plankton species
- Phylogenetically derived abundances of *Ostreococcus* clades



E_j = Evenness at location j

$$E_j = - \frac{\sum_i p_i \log(p_i)}{\log(S_j)}$$

p_i = relative species abundances
 S = # of species



CONCLUSIONS

- The concept of betweenness can be extended from the network context to fluid flows and dynamical systems, by relating it to Lyapunov exponents: **Lagrangian betweenness**.
- Locations with large Lagrangian betweenness identify circulation **bottlenecks**: regions in which flow is constrained: fluid parcels from different origins first converge and then diverge.
- **High betweenness** regions are seen to be **hot spots of biodiversity**.

Ser-Giacomi, Baudena, Rossi, Follows, Clayton, Vasile, López, Hernández-García, Nature Comm. **12**:4935 (2021)

<http://ifisc.uib-csic.es/publications/>



Grant PID2021-123352OB-C32 funded by MCIN/AEI/10.13039/501100011033 and by ERDF A way of making Europe